# Kinematics & Dynamics of Linkages

## Lecture 09 - Vectors

Ali H. Ammouri

# Chapter Objectives

1. Differentiate between a scalar quantity and a vector.
2. Apply trigonometry principles to right and general triangles
3. Determine the resultant of two vectors, using both graphical and analytical methods.
4. Resolve vector quantities into components in the horizontal and vertical directions.
5. Perform vector operations

# Introduction

- Mechanism analysis involves manipulating vector quantities.

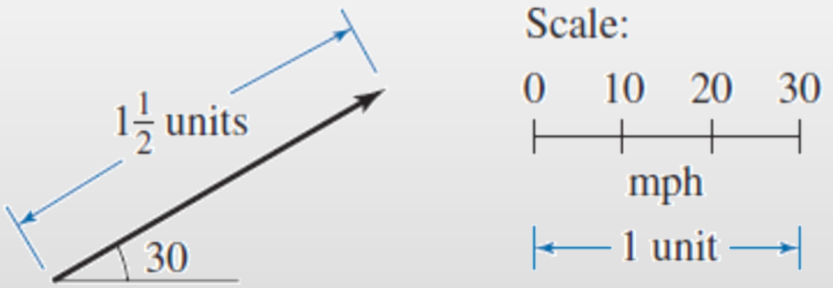
- Displacement, velocity, acceleration, and force are the primary performance characteristics of a mechanism, and are all vectors.

- Before working with mechanisms. A thorough revision of vectors and vector manipulation is needed

# Scalar VS Vector

- **Scalar:** quantity that is defined by stating only the magnitude:
  Example: a dozen of donuts

- **Vector:** a magnitude and direction is needed
  Example: the train is traveling at 50 mph in a northerly direction



# Graphical Vector Analysis

- A graphical approach to analysis involves drawing scaled lines at specific angles.
- To achieve results that are consistent with analytical techniques, accuracy must be a major objective.
- Using CAD tools made graphical techniques more reliable

# CAD knowledge required

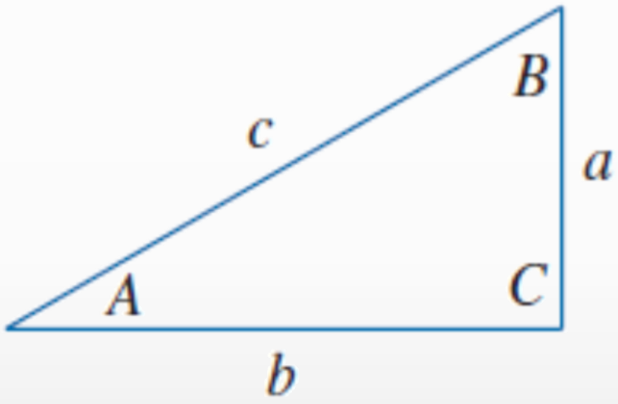
- Draw lines at a specified length and angle;
- Insert lines, perpendicular to existing lines;
- Extend existing lines to the intersection of another line;
- Trim lines at the intersection of another line;
- Draw arcs, centered at a specified point, with a specified radius;
- Locate the intersection of two arcs;
- Measure the length of existing lines;
- Measure the included angle between two lines.

# Trigonometry of a Right Triangle

$$\sin\angle B = \frac{b}{c}$$

$$\cos\angle B = \frac{a}{c}$$

$$\tan\angle B = \frac{b}{a}$$

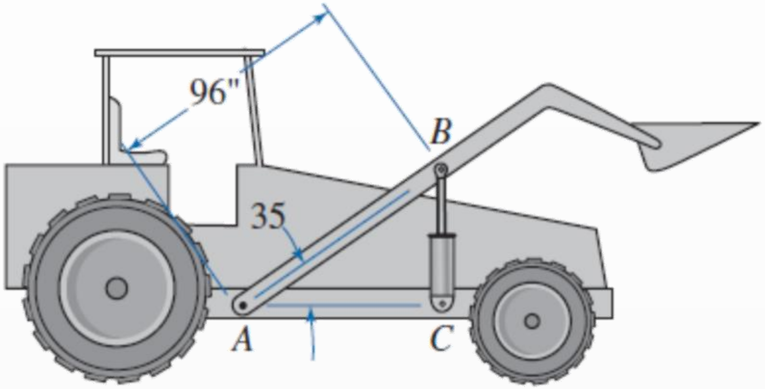



$$a^2 + b^2 = c^2$$

# Example 3.1

How much should the cylinder arm open to achieve the desired excavator position?

How far should the cylinder be mounted away from the arm pivot?




$$\sin\angle A = \frac{\text{opposite side}}{\text{hypotenuse}}$$

$$\sin 35^\circ = \frac{BC}{(96 \text{ in.})}$$

$$BC = (96 \text{ in.}) \sin 35^\circ = 55.06 \text{ in.}$$

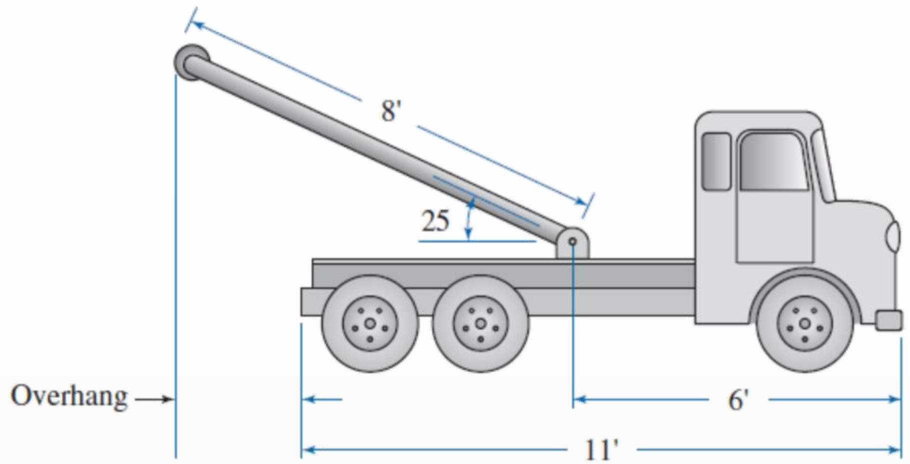
$$\cos\angle A = \frac{\text{adjacent side}}{\text{hypotenuse}}$$

$$\cos 35^\circ = \frac{AC}{(96 \text{ in.})}$$

$$AC = (96 \text{ in.}) \cos 35^\circ = 78.64 \text{ in.}$$

# Example 3.2

What is the horizontal distance that the boom extends from the truck




$$\cos 25^\circ = \frac{\text{horizontal projection}}{(8\text{ ft})}$$

$$\text{horizontal projection} = (8\text{ ft})\cos 25^\circ = 7.25\text{ ft}$$

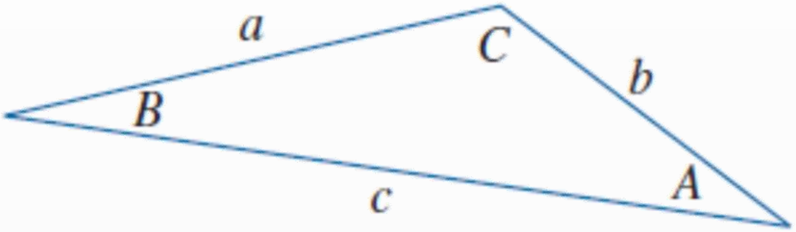
$$6\text{ ft} + 7.25\text{ ft} = 13.25\text{ ft}$$

$$13.25\text{ ft} - 11\text{ ft} = 2.25\text{ ft}$$


LAU

# Trigonometry of an Oblique Triangle

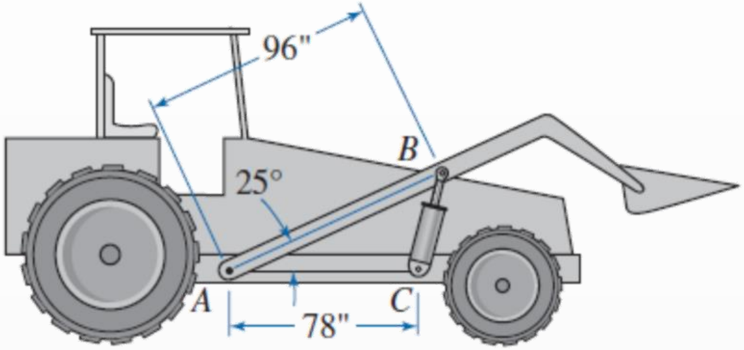
$$\frac{a}{\sin\angle A} = \frac{b}{\sin\angle B} = \frac{c}{\sin\angle C}$$




$$c^2 = a^2 + b^2 - 2ab \cos\angle C$$

# Example 3.3

Determine the required length of the cylinder to orient arm AB in the shown configuration




$$BC = \sqrt{AC^2 + AB^2 - 2(AC)(AB)\cos\angle BAC}$$

$$= \sqrt{(78\,\text{in.})^2 + (96\,\text{in.})^2 - 2(78\,\text{in.})(96\,\text{in.})\cos 25^\circ}$$

$$= 41.55\,\text{in.}$$

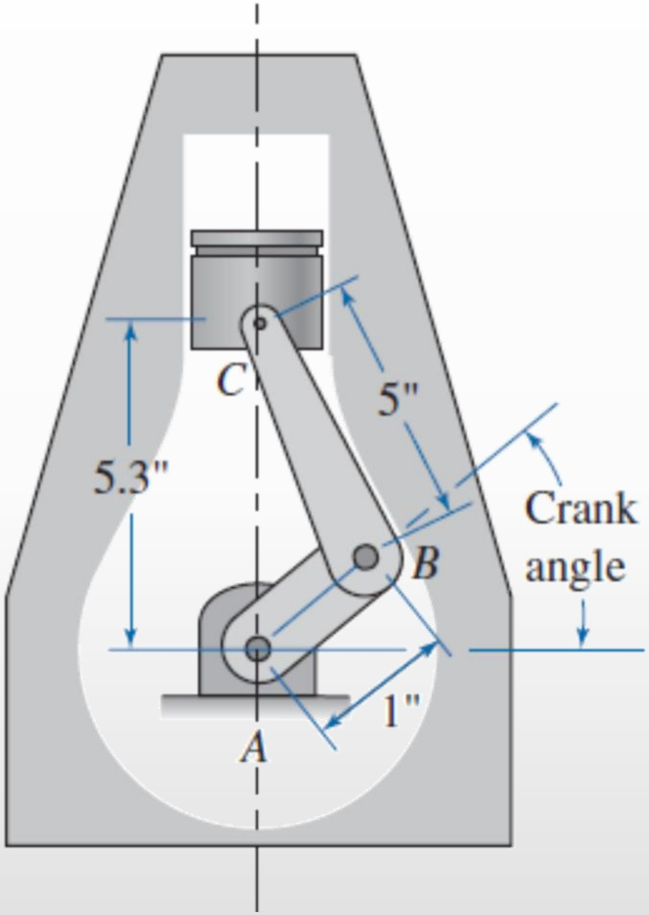
# Example 3.4

Determine the crank angle as shown in the figure.

$$\angle BAC = \cos^{-1}\left\{\frac{AC^2 + AB^2 - BC^2}{2(AC)(AB)}\right\}$$

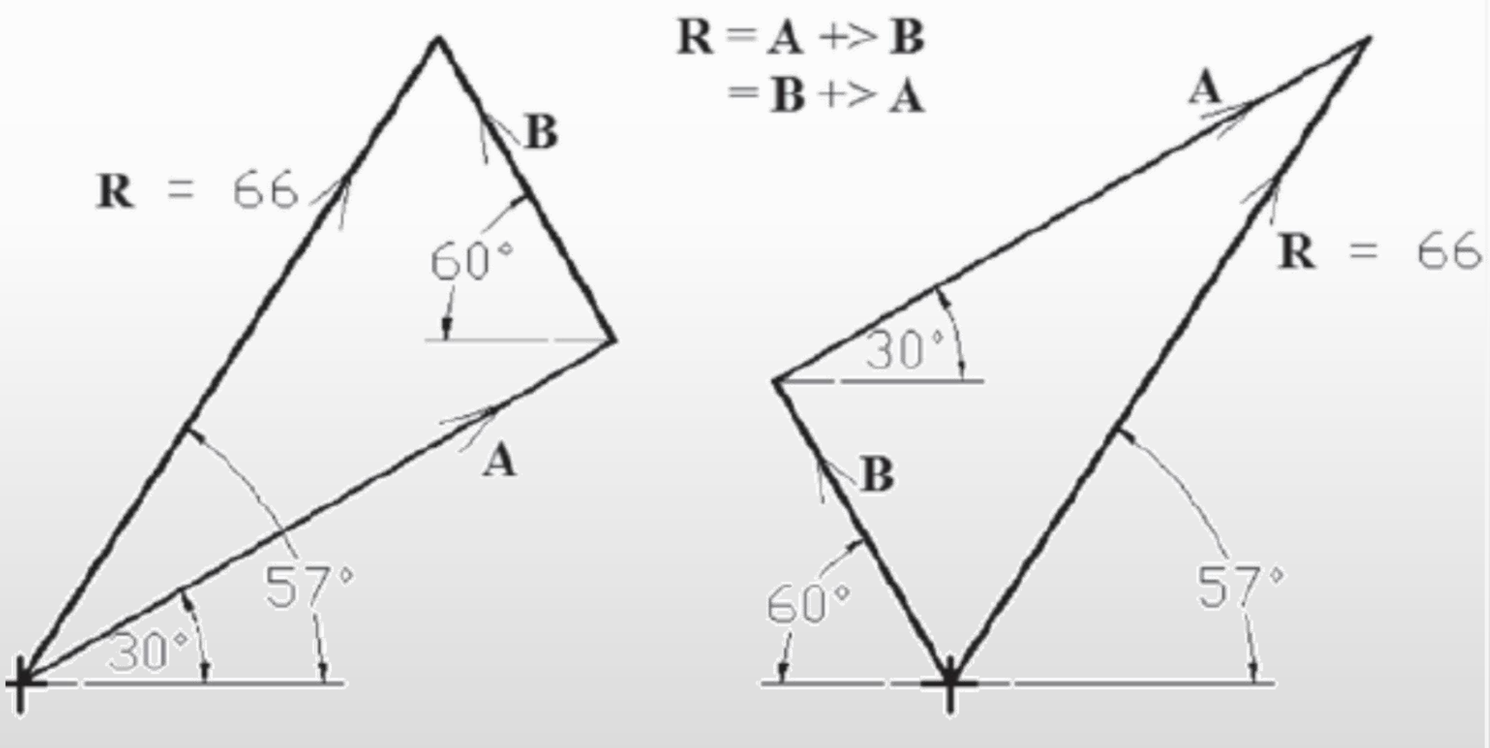
$$= \cos^{-1}\left\{\frac{(5.3\,\text{in.})^2 + (1\,\text{in.})^2 - (5\,\text{in.})^2}{2(5.3\,\text{in.})(1\,\text{in.})}\right\} = 67.3^\circ$$

$$\text{Crank angle} = 90^\circ - 67.3^\circ = 22.7^\circ$$

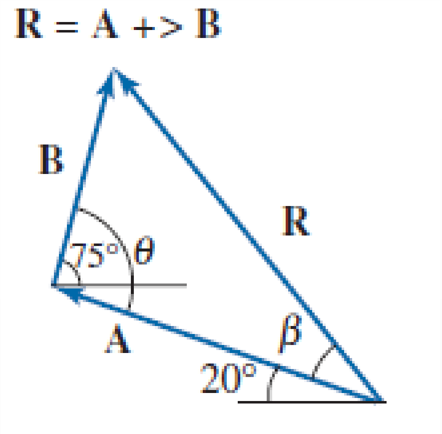


# Graphical Vector Addition

Graphically determine the effect of velocity vectors A:59<30 and B:30<120



# Analytical Vector Addition (+>)




Determine the resultant acceleration vector R

$$\theta = 20^\circ + 75^\circ = 95^\circ$$

$$R = \sqrt{A^2 + B^2 - 2AB\cos\theta}$$

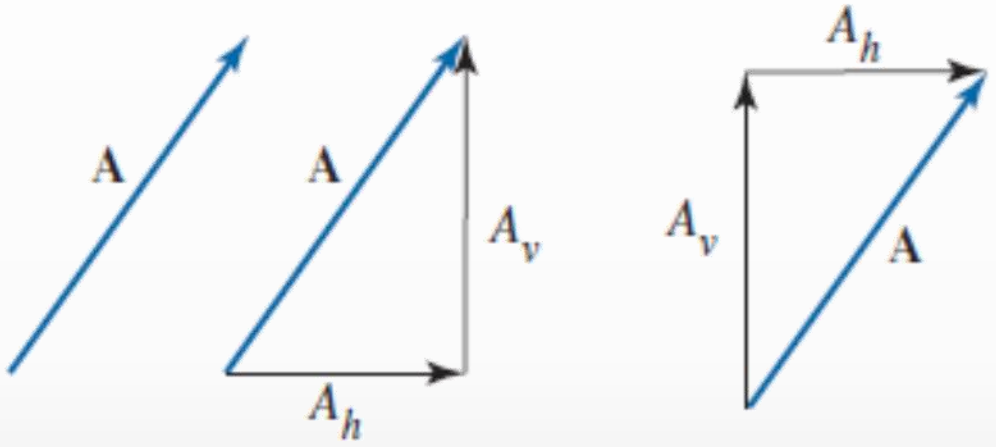
$$= \sqrt{(46\,\text{ft/s}^2)^2 + (23\,\text{ft/s}^2)^2 - 2(46\,\text{ft/s}^2)(23\,\text{ft/s}^2)\{\cos 9^\circ\}} = 53.19\ \text{ft/s}^2$$

$$\beta = \sin^{-1}\left\{\left(\frac{B}{R}\right)\sin\theta\right\}$$

$$= \sin^{-1}\left\{\frac{(23\,\text{ft/s}^2)}{(53.19\,\text{ft/s}^2)\sin 95^\circ}\right\} = 25.5^\circ$$

$$R = 53.19\ \text{ft./s}^2\ \angle 134.5^\circ$$

# Vector Components




$$A_h = A \cos \theta_x$$

$$A_v = A \sin \theta_x$$

# Analytical Vector Addition – Component Method

$$R_v = A_v + B_v + C_v + D_v + \ \ldots$$

$$R_h = A_h + B_h + C_h + D_h + \ \ldots$$

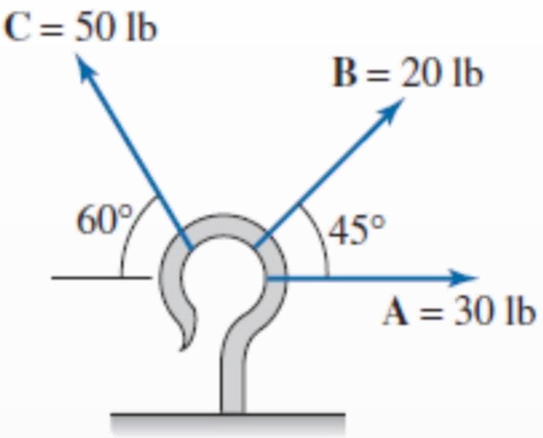
$$R = \sqrt{R_h^2 + R_v^2}$$

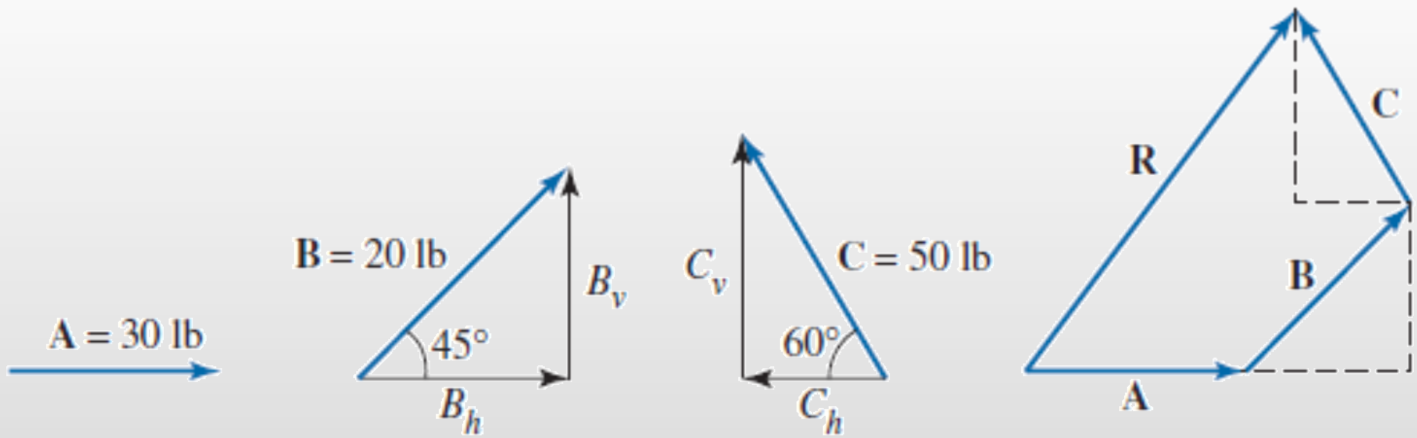
$$\theta_x = \tan^{-1}\left(\frac{R_v}{R_h}\right)$$

# Example 3.9

- Find the resultant force acting on the hook

- Components acting on the hook



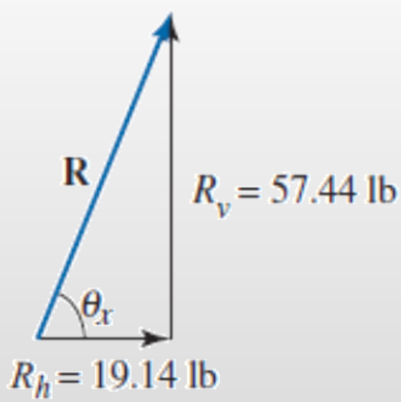



# Example 3.9 – Solutions

| Vector | Reference Angle $\theta_x$ | $h$ component (lb) $F_h = F \cos \theta_x$ | $v$ component (lb) $F_v = F \sin \theta_x$ |
|---|---|---|---|
| A | 0° | $A_h = (30)\cos 0° = +30$ lb | $A_v = (30)\sin 0° = 0$ |
| B | 45° | $B_h = (20)\cos 45° = +14.14$ lb | $B_v = (20)\sin 45° = +14.14$ lb |
| C | 120° | $C_h = (50)\cos 120° = -25$ lb | $C_v = (50)\sin 120° = +43.30$ lb |
| | | $R_h = 19.14$ | $R_v = 57.44$ |

$R_h = 19.14$ lb.

$R_v = 57.44$ lb.



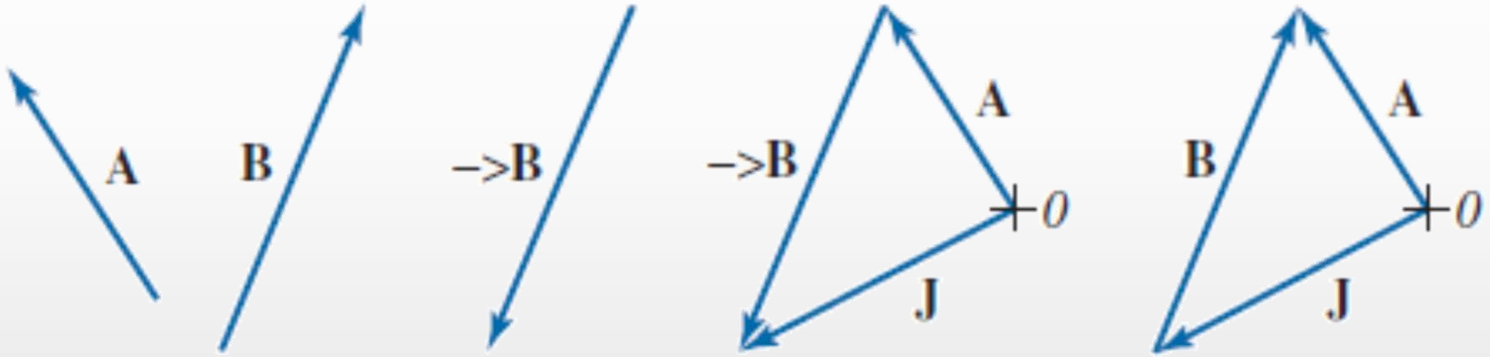

$$R = \sqrt{R_h^2 + R_v^2}$$
$$= \sqrt{(19.14\,\text{lb})^2 + (57.44\,\text{lb})^2} = 60.54\,\text{lb}$$

$$\theta_x = \tan^{-1}\left(\frac{R_v}{R_h}\right) = \tan^{-1}\left(\frac{57.44\text{ lb}}{19.14\text{ lb}}\right) = 71.6^{\circ}$$

$$\mathbf{R} = 60.54\text{ lb. } \angle 71.6^{\circ}$$

# Vector Subtraction (->)

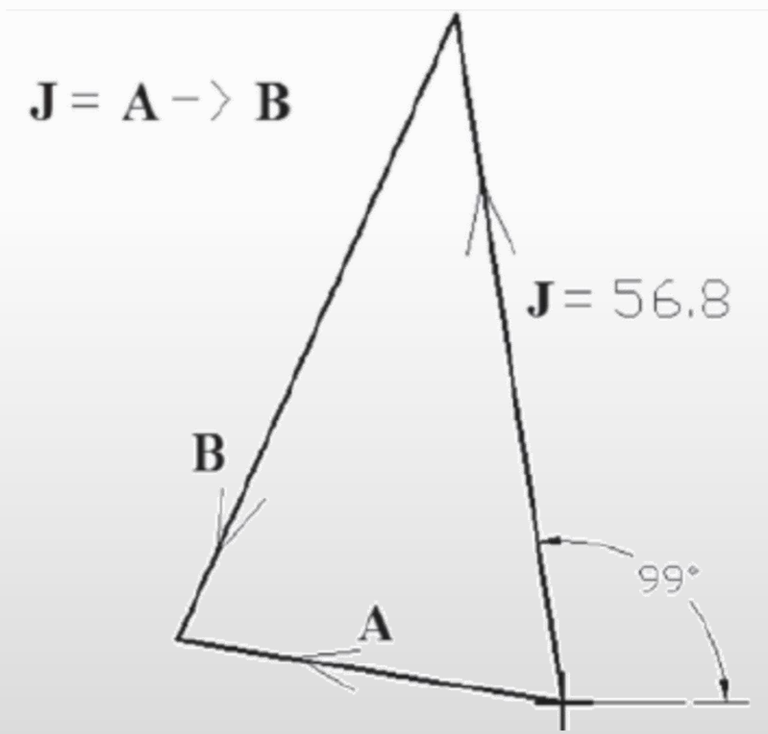
Adds a vector that has same magnitude but opposite direction




J = A –> B = A +> (–>B)

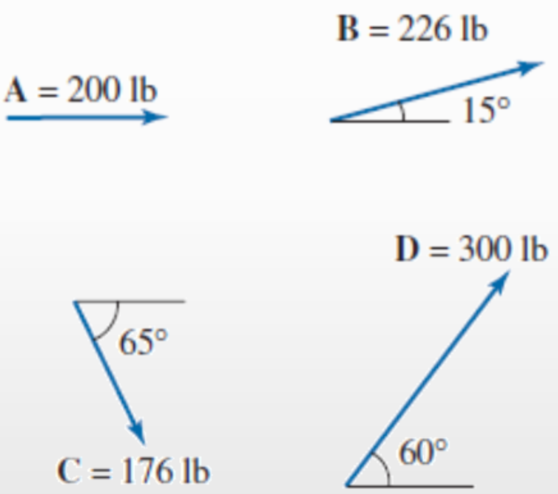
# Graphical Vector Subtraction

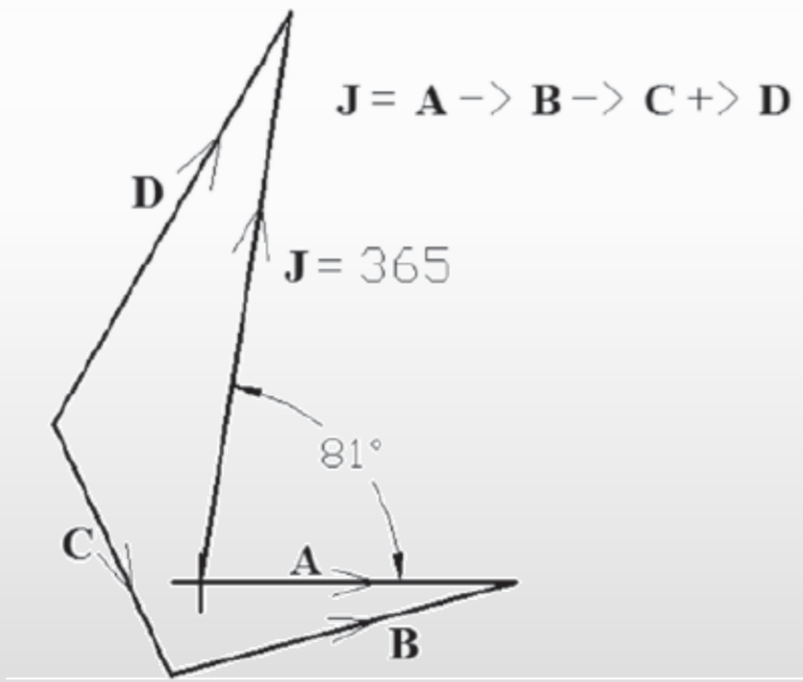
Graphically determine the effect of velocity vectors A:32<171 and B:56<294



# Example 3.11

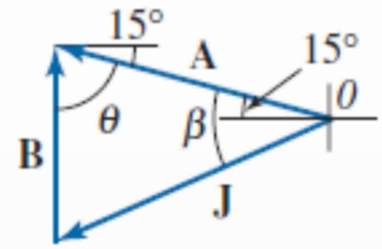
Graphically determine the result, J = A –> B –> C +> D, of the force vectors as shown in Figure



# Analytical Vector Subtraction (->)

Analytically determine the vector J




$$J = \sqrt{A^2 + B^2 - 2AB\cos\theta}$$

$$= \sqrt{(15\ \text{ft/s}^2)^2 + (10\ \text{ft/s}^2)^2 - 2(15\ \text{ft/s}^2)(10\ \text{ft/s}^2)\cos 75^\circ} = 15.73\ \text{ft/s}^2$$

$$\beta = \sin^{-1}\left\{\left(\frac{B}{J}\right)\sin\theta\right\}$$

$$= \sin^{-1}\left\{\frac{10\ \text{ft/s}^2}{15.73\ \text{ft/s}^2}\sin 75\right\} = 37.9^\circ$$

$$\mathbf{J} = 15.73\ \text{ft/s}^2 \;\; 22.9^\circ \swarrow$$

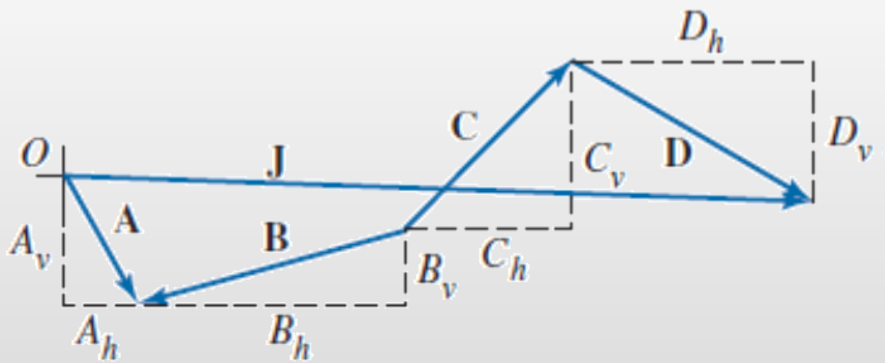
# Analytical Vector Subtraction – Component Method

$$\mathbf{J} = \mathbf{A} +> \mathbf{B} -> \mathbf{C} +> \mathbf{D} +> \ldots$$

$$J_h = A_h + B_h - C_h + D_h + \ \ldots$$
$$J_v = A_v + B_v - C_v + D_v + \ \ldots$$

# Example 3.13

Analytically determine the result J = A –> B +> C +> D for the velocity vectors shown in Figure




LAU

# Example 3.13 – Solution

| Vector | Reference Angle $\theta_x$ | $h$ component (ft/s) $V_h = V\cos\theta_x$ | $v$ component (ft/s) $V_v = V\sin\theta_x$ |
|---|---|---|---|
| A | 300° | +3.00 | −5.19 |
| B | 195° | −11.59 | −3.11 |
| C | 45° | +5.66 | +5.66 |
| D | 330° | +8.66 | −5.00 |

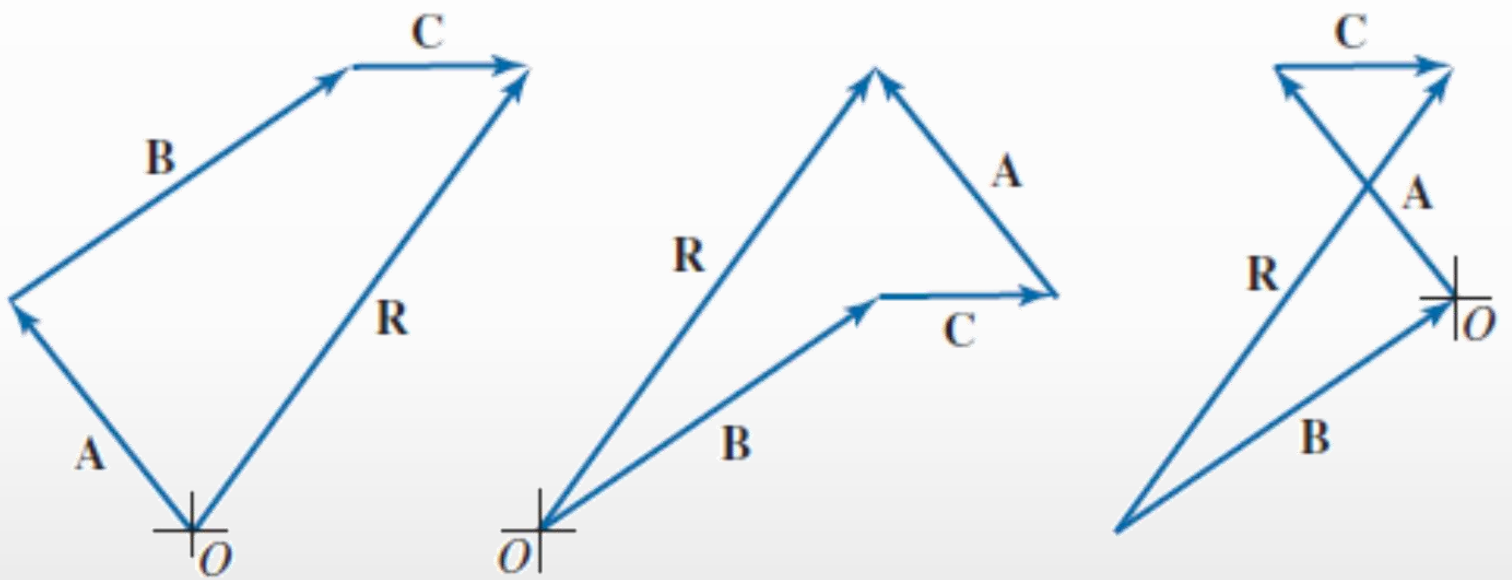
$$J_h = A_h - B_h + C_h + D_h$$
$$= (+3.0) - (-11.59) + (+5.66) + (+8.66) = +\ 28.91 \text{ ft/s}$$
$$J_v = A_v - B_v + C_v + D_v$$
$$= (-5.19) - (-3.11) + (+5.66) + (-5.00) = -1.42 \text{ ft/s}$$

$$J = 28.94 \text{ ft/s} \angle 2.8^\circ$$

$$J = \sqrt{J_h^2 + J_v^2}$$
$$= \sqrt{(28.91 \text{ ft/s})^2 + (-1.42 \text{ ft/s})^2} = 28.94 \text{ ft/s}$$

$$\theta_x = \tan^{-1}\left(\frac{J_v}{J_h}\right) = \tan^{-1}\left(\frac{-1.42 \text{ ft/s}}{28.91 \text{ ft/s}}\right) = -2.8^\circ$$

# Vector Equations



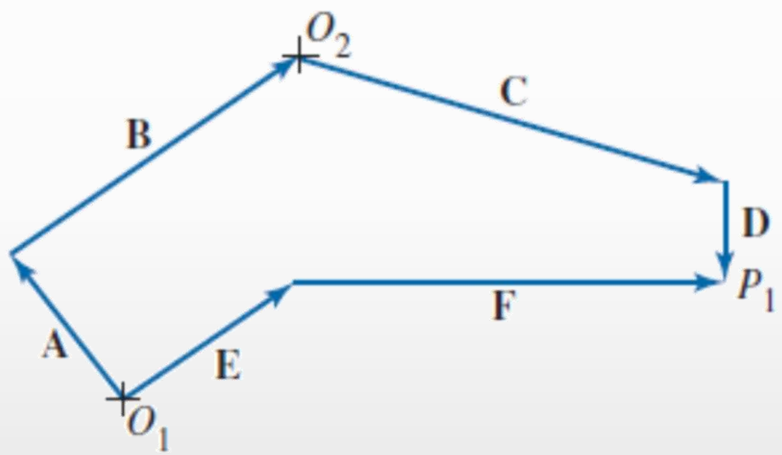

(a) A +> B +> C = R

(b) B +> C = R –> A

(c) A +> C = –> B +> R

# Example 3.14

Write a vector equation for the arrangement of vectors shown




$$O_1P_1 = A +> B +> C +> D = E +> F$$

# Applications of Vector Equations

- For problems where the magnitude of two vectors in an equation must be determined, the equation should be rearranged so that one unknown vector is the last term on each side of the equation.

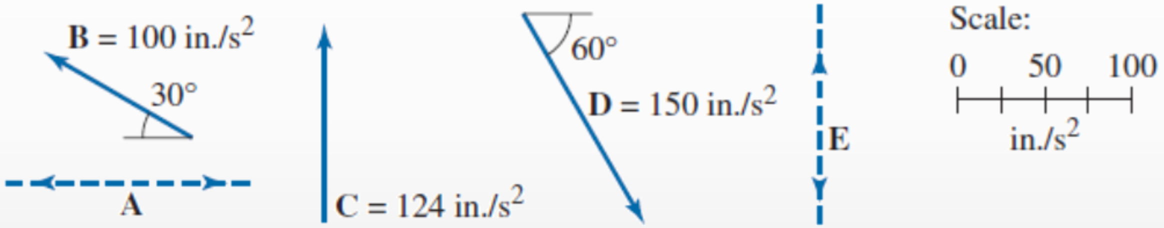
$$\mathbf{C} +> \mathbf{B} = \mathbf{D} +> \mathbf{E} +> \mathbf{A}$$

- To graphically solve this problem, the known vectors on each side of the equation are placed tip-to-tail (or tip to- tip if the vectors are subtracted) starting from a common origin.

# Example 3.16

$$A +> B +> C = D +> E$$

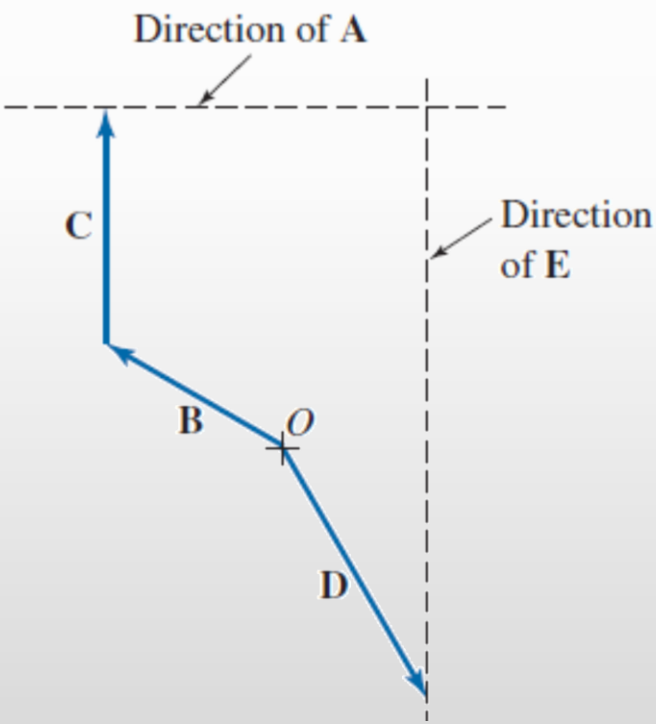
The directions for vectors **A, B, C, D,** and **E** are known, and the magnitudes of vectors **B, C,** and **D** are also known (Figure 3.35). Graphically determine the magnitudes of vectors **A** and **E.**

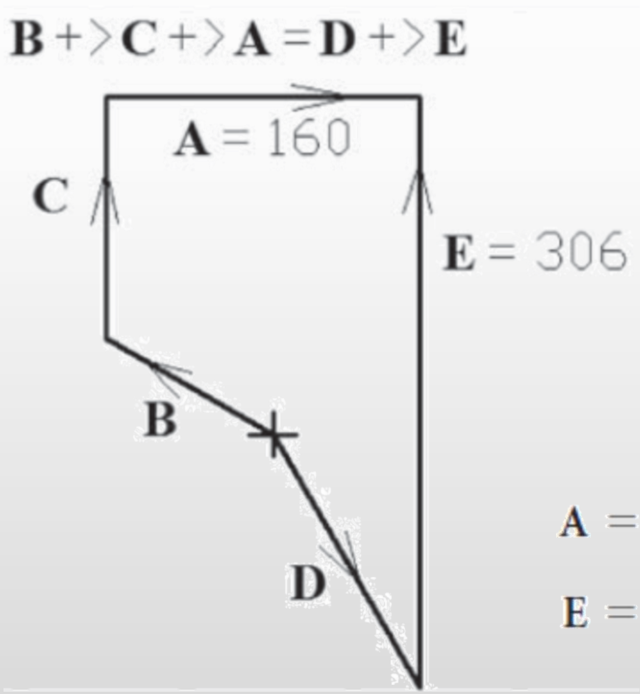



Step 1:

# Example 3.16 – Solution

B +> C +> A = D +> E




$A = 160\ \text{in./s}^2 \rightarrow$

$E = 306\ \text{in./s}^2 \uparrow$

# Analytical determination of Vector Magnitudes

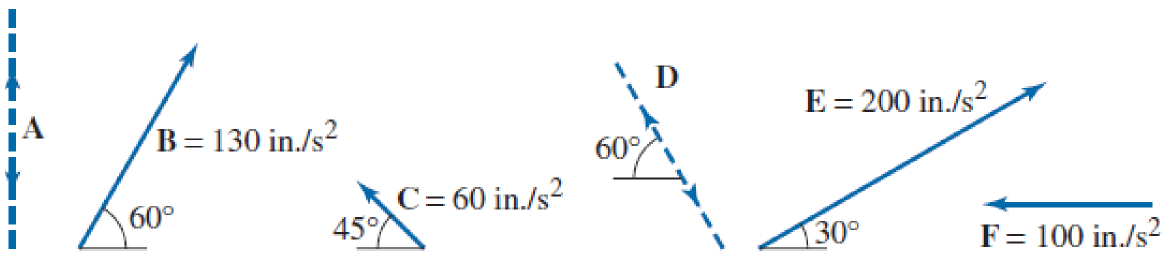
- An analytical method can also be used to determine the magnitude of two vectors in an equation. In these cases, the horizontal and vertical components of all vectors should be determined

- The horizontal components of the vectors must adhere to the original vector equation. Likewise, the vertical components must adhere to the vector equation. Thus, two algebraic equations are formed and two unknown magnitudes must be determined. Solving the two equations simultaneously yields the desired results

# Example 3.18

$$\mathbf{A} +> \mathbf{B} -> \mathbf{C} +> \mathbf{D} = \mathbf{E} +> \mathbf{F}$$

The directions for vectors **A**, **B**, **C**, **D**, **E**, and **F** are known, and the magnitudes of vectors **B**, **C**, **E**, and **F** are also known, as shown in Figure 3.39. Analytically solve for the magnitudes of vectors **A** and **D**.



# Example 3.18 – Solution

| Vector | Reference Angle $\theta_x$ | $h$ component (in./s$^2$) $a_h = a \cos\theta_x$ | $v$ component (in./s$^2$) $a_v = a \sin\theta_x$ |
|---|---|---|---|
| A | 90° | 0 | +A |
| B | 60° | +65.0 | +112.6 |
| C | 135° | −42.4 | +42.4 |
| D | 300° | +.500D | −.866$D$ |
| E | 30° | +173.2 | +100 |
| F | 180° | −100 | 0 |

$$A +> B -> C +> D = E +> F$$

Horizontal Components:

$$A_h + B_h - C_h + D_h = E_h + F_h$$

$$(0) + (+65.0) - (-42.4) + (+0.500\,D) = (+173.2) + (-100.0)$$

# Example 3.18 – Solution

Vertical Components:

$$A_v + B_v - C_v + D_v = E_v + F_v$$

$$(+A) + (+112.6) - (42.4) + (-0.866\,D) = (+100.0) + (0)$$

Solving both equations yield:

$$D = -68.4 \text{ in./s}^2$$

$$A = -29.4 \text{ in./s}^2$$

$$\mathbf{A} = 29.4 \text{ in./s}^2 \downarrow$$

$$\mathbf{D} = 68.4 \text{ in./s}^2 \; 60^\circ \nwarrow$$